Efficient Heuristics for MRP Lot Sizing with Variable Production/Purchasing Costs*

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ABSTRACT

Two heuristics based on branch and bound (B&B) are developed to solve closed-loop material requirements planning (MRP) lot-sizing problems that have general product structures and variable costs. A "look ahead method" (LAM) heuristic allows for variable production/purchasing costs and uses a single-level B&B procedure to rapidly improve lower bound values; thus, LAM efficiently uses computer-storage capacity and allows solution of larger problems. The "total average modification" (TAM) heuristic uses B&B, applied level by level, and modified setup and carrying costs to solve the variable production/purchasing costs MRP lot-sizing problem. LAM and TAM are tested on problems and compared to heuristics in the literature. TAM may be used to solve large MRP lot-sizing problems encountered in practice.

Subject Areas: Heuristics, Inventory Management, Material Requirements Planning, and Production/Operations Management.

INTRODUCTION

It is fairly well known that lot-sizing decisions in material requirements planning (MRP) systems require the solution to a very large combinational problem, amenable to solution by heuristic methods. Many lot-sizing heuristics used on multilevel product structures assume constant demand patterns (economic order quantity), constant production/purchasing costs [17] [24] [3], or single-level product structures [29] [19]. However, the authors know of no heuristics developed for what may be the most prevalent lot-sizing problem found in the manufacturing industry today: the MRP multiperiod, multilevel, multicomponent, multiparent, variable production/purchasing costs problem.

Many optimal and heuristic single-level lot-sizing techniques with deterministic time varying demand have been presented and tested in the literature [1] [2] [4] [5] [11] [13] [25] [29]. Quantity discounts have also been included as an extension to the single-level problem [14] [15] [26]. The single-level product structure, however, rarely represents the complexity required when manufacturing products. The use of a serial product structure (i.e., multilevel, single component, single parent) or an assembly product structure (i.e., multilevel, multicomponent, single parent) may not be appropriate in manufacturing either. The general product structure (i.e., multilevel, multicomponent, multiparent) used in this paper is often required.

Recently, Gaither [9], Jacobs and Khumawala [12], and Veral and LaForge [28] tested many single-level heuristics (i.e., lot for lot, economic order quantity, periodic order quantity, least unit cost, least total cost, part period balancing, etc.) in a multilevel product structure environment in comparison with the Wagner and

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Whitin (WW) algorithm [29]. Based on cost performance, the results identify WW as generally superior. Many other studies testing single-level heuristics in a multilevel product structure may be found in the literature [18] [21] [25]. However, studies on heuristics designed especially for the multilevel lot-sizing problem in an MRP environment are addressed by relatively few researchers.

McLaren and Whybark (MW) [17] specifically addressed the multilevel problem and develop a heuristic that inflates the setup costs of parent items based only on the items' immediate component's setup costs. The adjusted setup cost is used in place of the original in the single-level, periodic order quantity (POQ) lot-sizing procedure. Rehmani and Steinberg [24] extended MW's research by modifying each parent's economic part period (EPP) calculation by each of their immediate component's EPP. They proposed and tested four modifications to an item's EPP and used them with the least total cost (LTC) single-level lot-sizing technique. Graves [10] presented a multipass heuristic that continues to revise and improve the current schedule in an iterative fashion. Graves' procedure starts with a single-level WW solution that is collapsed into other stages based on the current schedule. Recently, Billington, McClain, and Thomas [3] presented a Lagrangian relaxation heuristic method to solve a multilevel lot-sizing problem with a single bottleneck facility.

The use of closed-loop MRP systems [31] to plan and control manufacturing processes allows the use of a general product structure model and requires that solution heuristics make the most efficient component quantity and timing decisions. Lot sizing in this research incorporates many conditions found in manufacturing, resulting in perhaps the most representative MRP lot-sizing problem of those found in the literature. The heuristics presented in this paper are, however, easy to implement because they follow standard MRP netting requirements and planned order release calculations [19] when solving the MRP lot-sizing problem with variable costs. Variable production/purchasing cost is the newest feature of this general case, heuristic lot-sizing research. "All units" quantity discounts, where all the units' prices decline relative to increased purchase quantities once a price break is reached, are available in the model to represent variable purchasing costs [20] [30]. For those items produced in-house, variable production costs include capacity constraints for an item during any one period. Once the normal production limit is reached, increased per-unit variable production costs may be incurred due to overtime or subcontracting costs.

An integer, piecewise-linear programming model that incorporates the above MRP lot-sizing features can be found in Prentis and Khumawala [22]. Branch and bound (B&B) (briefly reviewed in the following section) has been used to optimally solve this problem using a path-dependent, lower-bound procedure where the costs on new lot-sizing decisions are added to the preceding node's lower bound [22]. Two heuristics, the look ahead method (LAM) and total average modification (TAM), are developed based on the B&B algorithm. LAM and TAM are tested on those problems in the literature with "good" results.

THE LOOK AHEAD METHOD HEURISTIC

In spite of the B&B procedure's efficiency, the number of nodes created in B&B grows exponentially with problem size and becomes a limiting constraint. The lower-bound value does not begin to approach the upper-bound value until many nodes have been created. Most of these nodes never lead to an optimal solution but are stored during the B&B process, thus taxing computer-storage capacity. For the MRP lot-sizing problem with variable production/purchasing costs, the largest problems solved using B&B were comprised of five items over 12 periods, each with different demand and cost patterns, and required a maximum of 180 minutes of computer time [22].

The following example demonstrates and briefly reviews the logic of B&B and serves as an introduction to the LAM heuristic procedure. The demand requirements for end items 1 and 2 are listed in the master production schedule (MPS) (Table 1).

The bill of material (BOM) is for a multilevel, multicomponent, multiparent general product structure problem (Figure 1). Item 1 is produced with 2 units of component 3 and 1 unit of component 4. Item 2 is processed from 1 unit of component 4. The branching strategy begins first with the lowest-numbered end item (i.e., item 1) and progresses sequentially to the highest-numbered item located at the lowest level (level 2) in the BOM (i.e., 1 to 2 to 3 to 4). The inventory records file (IRF) includes the setup and carrying costs for each item as listed below:

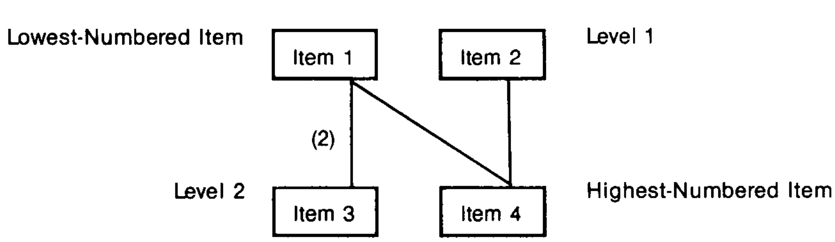
Setup costs	Carrying costs
items 1, 2=$45	items 1, 2=$2/period/unit
items 3, 4=$20	items 3, 4=$1/period/unit

During the B&B branching process the problem is continually partitioned into reduced subsets which are identified as branches and associated nodes. A lower bound is calculated for each new node after each partitioning. Subsets with a lower bound higher than or equal to a known feasible solution are removed (i.e., pruned) from the available solution set and discarded. The lower bound calculation at each node uses information associated only with that node's branching path (i.e., path-dependent through the B&B tree). Figure 2 shows the B&B solution to this example problem. Lot sizes are shown above the nodes for each item and order periods are denoted after dashed lines. Nodes not leading to an optimal solution are pruned (≠). Computations for a few intermediate and representative nodes, identified in Figure 2, follow.

Table 1: Master production schedule.

	Period			
Product	1	2	3	4
Item 1	10	—	30	—
Item 2	—	40	—	5

Figure 1: Bill of material.



The initial lower bound cost (IL) is first calculated for our example problem to include one setup for each item. Therefore, the IL for node 0 is 130 (i.e., 45+45+20+20). Branching and bounding begins with item 1 in period 1. The two options are (1) to produce 10 units to meet demand in period 1, thus requiring one more setup of item 1; or (2) to produce 40 units to meet demand for periods 1 and 3, thus requiring 30 units of item 1 to be carried two periods. Based on these decisions, lower bound costs (L) at nodes 1 and 2 are $L=130+45=175$ and $L=130+(30\times2\times2)=250$, respectively. Each of the lower bound values are then compared to a feasible solution's upper bound value (a lot-for-lot solution to this problem is 300). The node may be pruned if the lower bound is greater than or equal to the upper bound value. Since the lower bounds for nodes 1 and 2 are less than the upper bound, B&B continues along both B&B tree paths. The lower bound value along the optimal path from node 1 to node 9 remains $L=175$. At node 12, the lot size for item 2 in period 2 is 45, which requires carrying 5 units of item 2 for two periods. Therefore, node 12's lower bound cost is $L=175+(5\times2\times2)=195$. Continuing from node 12 to node 22, the lot size for item 3 is 80 units in period 1. The resulting lower bound value for node 22 is $L=195+(60\times1\times2)=315$, which is greater than the upper bound value and is therefore pruned ($\neq$). The optimal planned order releases (POR) for each item in the example problem are shown in Figure 3.

The LAM heuristic modifies the B&B procedure by using a B&B single-level optimizing subroutine to increase the lower bound values faster. The following notations are used for LAM:

Opt i = Single-level B&B solution of item i using a selected POR.

L_n = Lower bound cost at a node using B&B which is path-dependent through the B&B tree.

ϕ = Set of all higher-numbered items solved for on a single-level basis using POR along the path with the lowest bound when the item was first addressed.

The lower bound cost calculation for LAM (L_l) at a node is now

$$L_l=L_n+\sum_{i\in\phi}\text{Opt } i \qquad \text{for } \forall\, i,t.$$

When B&B is used, the next node selected is often a previously addressed item that has been partially enumerated. What has been learned about the higher-numbered items (e.g., items 2 and 3) is not taken into account when calculating the lower bounds for lower-numbered items (e.g., item 1) (see Figure 2). If the higher-numbered item's B&B single-level solution (i.e., Opt i, which is a B&B solution for only one item's requirements) is added to the B&B lower bound values, then pruning can be enhanced. This is the central idea behind the development of the LAM heuristic.

A B&B single-level optimizing subroutine allows for variable production/purchasing costs and is used when each higher-numbered item is encountered. The gross requirements for a component, associated with the branching path having the least lower bound when this component first comes into solution, is employed and solved optimally (using B&B) on a single-level basis (Opt i). Any time the lower bound node selection drops back to a lower-numbered item, the lower bound calculation adds the Opt i solutions of all higher-numbered items already encountered and solved optimally on a single-level basis. Figure 2 shows a B&B solution for our

Figure 2: Pruning the B&B tree.

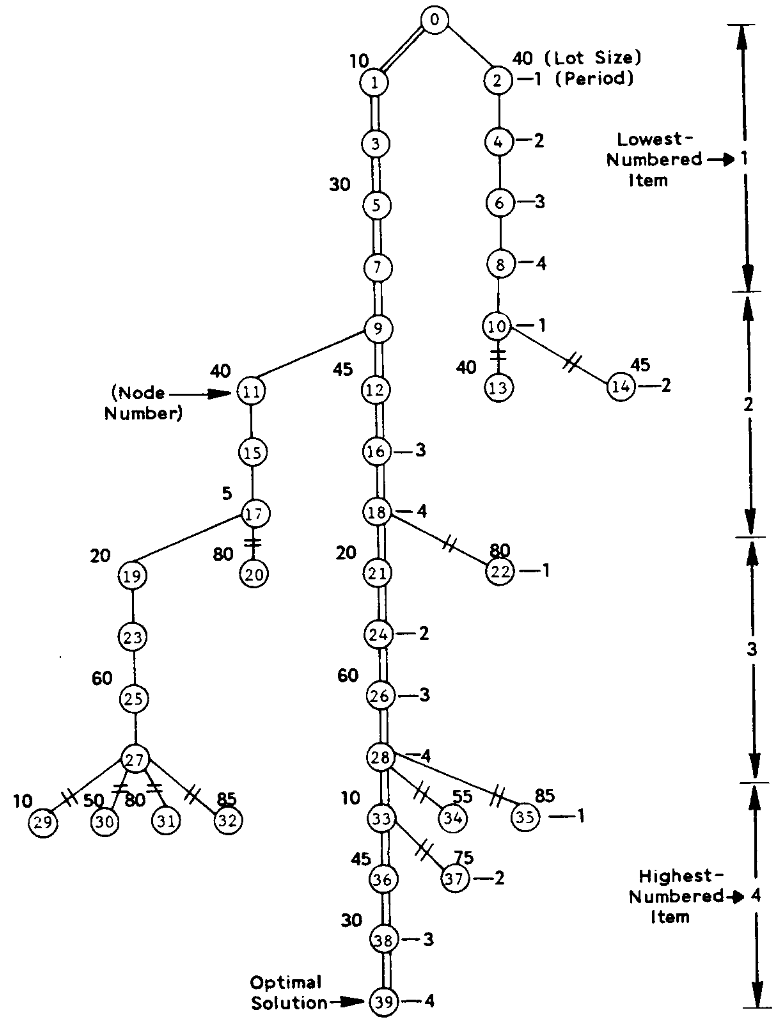


Figure 3: Final solution.

Level 1

Item 1

Period	1	2	3	4
GR	10		30	
POR	10		30	

Item 2

Period	1	2	3	4
GR		40		5
POR		45		

Level 2

Item 3 (2)

	1	2	3	4
GR	20		60	
POR	20		60	

Item 4

	1	2	3	4
GR	10	45	30	
POR	10	45	30	

Note: GR = gross requirements,
POR = planned order releases.

general product structure example problem. The B&B procedure generates a total of 39 nodes for this example problem's solution.

LAM begins by calculating the B&B single-level solution to end items 1 and 2, represented here by Opt 1 and Opt 2. Higher-numbered items' B&B single-level solutions (i.e., items 3 and 4) are calculated when the item is first addressed during the B&B process. Node 18 has the lowest lower bound cost ($L=195$) along the B&B path leading to item 3. Therefore, Opt 3 would be calculated using the B&B tree path of PORs of 10 and 30 in periods 1 and 3 for item 1 and 45 in period 2 for item 2. Item 2 is not a parent of item 3 so it would not be a factor in Opt 3's solution; however, it could be. If node 2 (representing item 1) was the next node selected, the lower bound value would include values of Opt 2+Opt 3; consequently, the resulting increased lower bound value might prune paths 2, 4, 6, 8, 10, 13, and 14 at node 2 rather than waiting until nodes 13 and 14 when using B&B. This LAM heuristic will not guarantee optimality. However, LAM does provide "good" solutions and reduces computer-storage space requirements for nodes when compared to B&B.

The number of nodes created are reduced by eliminating those that are highly unlikely to lead to an optimal solution. It is important to note that each B&B node's lower bound value (L_n) is calculated on its hierarchical product structure relationship (i.e., it is path-dependent through the B&B tree). The optimal single-level B&B (Opt i) solution is used only when the branching strategy selects a lower-numbered item from which to branch. Only those nodes with the highest lower bound values are eliminated while maintaining the B&B lot-sizing procedure. This eliminates excessive computer storage requirements and yet provides "good" solutions. The number of nodes created when using the LAM heuristic grows exponentially with problem size. Therefore, to solve industry-sized problems from the literature the TAM heuristic (presented next) is required.

THE TOTAL AVERAGE MODIFICATION HEURISTIC

TAM is based on heuristics in the literature [6] [16] [17] [23] [24] as well as on the B&B [22] solution technique. McLaren and Whybark [17] and McLaren [16] developed a heuristic to be used with single-level procedures on the multilevel lot-sizing problem. They adjusted the item's setup cost by summing the time-between-orders (TBO) ratio, multiplied by the component part's setup cost. Their procedure includes only the parent's immediate components on the next lower level in the BOM. Components after the next lower level are ignored when adjusting the parent's setup cost even though interaction of orders at these levels will affect the optimal decision.

Instead of adjusting only the setup cost, Rehmani and Steinberg [24] modified each item's EPP. Of the four variations studied, the simple average modification (SA) to the EPP calculation produced the best results; therefore, SA is used for comparison purposes in this study. Blackburn and Millen [6] developed heuristics for the assembly product structure (i.e., multilevel, multicomponent, single parent) problem. They used level-by-level, single pass algorithms (i.e., WW [29], Silver-Meal [27], part period balancing [19]) along with cost modifications in an attempt to incorporate the interdependencies of items among levels. They suggested that research be done on how to treat multiparent product structures. The solution of the general product structure model which incorporates the work done in [6], [17], and [24] is addressed next.

The approaches in [17] and [24] modify each parent's costs by the immediate components' costs on the next lower level in the BOM even though all lower level components interact with that upper level parent item. However, Blackburn and Millen [6] show that taking all components into account when calculating a parent's modified setup/ordering and echelon holding costs achieves better results than (1) when no cost modifications are used, and (2) when only the immediate components' costs are incorporated.

The TAM heuristic is designed to recognize that all components throughout the BOM affect a parent's lot size [21]. This is the interaction effect throughout all levels in the product structure that must be accounted for when developing an MRP lot-sizing heuristic. Therefore, TAM modifies a parent's setup/ordering and carrying costs using all of the item's components rather than just the parent item's immediate components.

Product structures with a degree of commonality greater than one (see [7] and [8]) will have multiple common components for a parent item. When modifying a parent's setup or carrying costs, a decision must be made as to how many times multiple components should be incorporated when requiring more than one of the same component. The unique multilevel component approach (i.e., each component is included once, regardless of the number of each component required for a parent) for TAM is based on the literature. Rehmani [23] tested multiple immediate components for parent items when calculating the weighted sum modification (WS) to the EPP calculation. Rehmani found that WS performs no better than the SA (which uses unique immediate components) and that WS may exaggerate order sizes. Therefore, the TAM heuristic takes into account all unique multilevel components for a parent item at all levels in the product structure's BOM. Thus, both the setup/ordering and carrying costs are modified based on the respective costs of all unique multilevel components. The following modified setup and carrying costs are used in place of the original F_i and C_{it} values for each item in the product structure:

$$F_i' = 1/R_i \left[F_i + \sum_{k \in A_i} F_k \right]$$

$$C_{it}' = 1/R_i \left[C_{it} + \sum_{k \in A_i} C_{kt} \right],$$

where

F_i = the fixed cost of setting up production or ordering for item i, regardless of quantity;
F_i' = modified setup/ordering cost;
C_{it} = cost of carrying one unit of item i in period t;
C_{it}' = modified carrying cost;
A_i = set of all unique components of item i;
R_i = the number of items in the set containing item i and A_i.

TAM uses modified setup and carrying costs based on all unique multilevel components throughout the BOM in conjunction with B&B applied level by level. This solves the general product structure, variable production/purchasing costs problem for each item and approximates the multilevel problem solution approach of B&B. In addition, TAM solves the MRP lot-sizing problem with scheduled receipts, on-hand inventory, independent demand at lower levels in the product structure, as well as "all units" quantity discounts and item capacity constraints.

The LAM and TAM heuristics are examined on a wide variety of test problems; their computational results are presented in the next section.

COMPUTATIONAL RESULTS

The computational experiments consist of evaluating the LAM and TAM heuristics by varying four factors. (1) Six demand patterns for independent demand items are used (level, increasing, decreasing, concave, convex, and lumpy) for the 12-period and 52-period problems (Exhibit 1 is representative of the 52-period problems). (2) Cost patterns are varied to evaluate procedures over different economic circumstances. Setup costs are varied throughout the product structure levels because setup costs for final assembly may be less than when fabricating components [22] [23]. (3) The number of product structure levels ranges from three to five levels. (4) Four degrees of commonality index (no, low, medium, high), as defined by Collier [7] [8] for the product structure design, are used. The number of items ranges from 5 to 62.

Exhibit 1: Demand patterns, 52 periods (adapted from Rehmani [23]).

Increasing	$d_{(i+4k)}=10+2k$, $i=1$ to 4; $k=0$ to 12.
Decreasing	$d_{(i+4k)}=34-2k$, $i=1$ to 4; $k=0$ to 12.
Concave	$d_i=8+i$, $i=1$ to 27; $d_k=62-k$, $k=28$ to 52.
Convex	$d_i=36-i$, $i=1$ to 27; $d_k=k-18$, $k=28$ to 52.
Level	$d_i=22$, $i=1$ to 52.
Lumpy	$d_i=\begin{cases} 40 & \text{for } i=5,\ 49; \\ 50 & \text{for } i=12,\ 41,\ 45; \\ 60 & \text{for } i=1,\ 19,\ 40; \\ 70 & \text{for } i=7,\ 15,\ 51; \\ 80 & \text{for } i=29,31; \\ 84 & \text{for } i=36; \\ 90 & \text{for } i=24,27; \\ 100 & \text{for } i=32; \\ 0 & \text{elsewhere, } i=1 \text{ to } 52. \end{cases}$

Average demand per period $(\bar{d})=22$

To solve the test problems, two FORTRAN 77 computer programs—one for LAM and the other for TAM—were written. The TAM heuristic is tested on the 52-period problems and compared to the following methods: (1) Wagner and Whitin (WW) [29], (2) McLaren and Whybark (MW) [17], and (3) Rehmani and Steinberg (RS) [24]. The percent savings of inventory costs for the various procedures are reported.

A cost index (CI) is used to facilitate comparisons of lot-sizing methods and results. The CI for the 12-period problems is based on B&B [22] because of the variable production/purchasing costs of these test problems. To prevent the common total production/purchasing costs from overwhelming the 12-period problem percentage results, these common costs have been removed from the TAM, LAM, and B&B solution values prior to calculating percentages. Thus, the most stringent comparison of TAM's effectiveness is achieved by basing percentages only on the costs presently under the production manager's control (i.e., setup/ordering, carrying, "all units" quantity discounts, and item capacity constraints). To calculate the 12-period problem's cost indices, the following formula is used:

$$CI = \frac{\text{Technique's cost}}{\text{B\&B cost}} \times 100.$$

For the 52-period problems, the cost indices are based on WW (for a representative comparison with other techniques in the literature) and is found by:

$$CI = \frac{\text{Technique's total cost}}{\text{WW total cost}} \times 100.$$

The results of the 12- and 52-period test problems from the literature follow.

The 12-period problems have a general product structure, "all units" quantity discounts, and item capacity constraints [22]. Table 2 presents 12 total cost results for each technique tested (i.e., B&B, LAM, and TAM). LAM achieved an optimal solution to the general product structure model with variable production/purchasing costs in every case. When compared with B&B for number of nodes used, LAM saved an average of 25 percent. Reduced node creation by LAM allows efficient use of computer storage capacity; thus, larger problems may now be solved. TAM gave solutions that cost, on average, only 3 percent more than the B&B solutions.

The large 52-period demand problems, with up to 62 items and five product structure levels, are taken from Rehmani [23]. Results from a total of 48 problems are reported for the TAM, RS, and MW techniques. These MRP lot-sizing problems incorporate the general product structure. Production/purchasing costs for individual items are zero (i.e., no "all units" quantity discounts or item capacity constraints are included in these test problems) to make a fair comparison to the less versatile multilevel heuristic techniques found in the literature (i.e., [17], [24], and [29]). The TAM heuristic proves superior on these problems compared to all other heuristics tested (Tables 3 through 5). Even without the variable costs complexity, the TAM heuristic saves approximately 18 percent when compared to WW when applied level by level. TAM also gives approximately an 8 percent cost savings when compared to RS or MW.

CONCLUSIONS

This research addresses the MRP lot-sizing problem in a manufacturing environment and develops heuristics that show good results. The look ahead method (LAM) and total average modification (TAM) heuristics are developed based on a variable production/purchasing costs MRP lot-sizing model and the branch and bound (B&B) solution procedure. The two heuristics, LAM and TAM, are tested for their computational efficiency compared to existing methods found in the current literature. On the 12-period problems tested, LAM proved optimal 100 percent of the time while reducing computer storage requirements by saving an average of 25 percent on the number of B&B nodes created. TAM averaged only 3 percent above optimal costs.

TAM was developed for use on larger (e.g., 52-period) problems. On the 52-period problems tested, the TAM heuristic averaged an 8 percent lower cost than the McLaren and Whybark (MW) [17] or Rehmani and Steinberg (RS) [24] heuristics and an 18 percent lower cost when compared to the Wagner and Whitin (WW) algorithm [29]. TAM has been tested using problems in the literature that have a wide variety of demand patterns, cost values, product structure commonalities,

Table 2: Cost indices (12 periods) for B&B, LAM, and TAM.

Cost Indices	Branch & Bound (B&B)[1]	Look Ahead Method (LAM)	Total Average Modification (TAM)
Example A			
Average	100	100	101
Range	*	100-100	100-103
Example B			
Average	100	100	104
Range	*	100-100	100-110

[1]From Prentis and Khumawala [22].
*Cost index based on B&B.

Table 3: 52-period results—Four levels.

Cost Indices	Wagner and Whitin [29]	Rehmani and Steinberg [24]	McLaren and Whybark [17]	Total Average Modification (TAM)
No commonality				
Average	100	90	88	85
Range	*	87-93	86-92	84-86
Low commonality				
Average	100	94	96	84
Range	*	91-102	90-103	83-86
Medium commonality				
Average	100	94	87	84
Range	*	81-108	81-92	80-91
High commonality				
Average	100	97	104	84
Range	*	90-107	91-108	79-88

*Cost index based on Wagner and Whitin [29].

Table 4: 52-period results—Five levels.

Cost Indices	Wagner and Whitin [29]	Rehmani and Steinberg [24]	McLaren and Whybark [17]	Total Average Modification (TAM)
No commonality				
Average	100	85	84	82
Range	*	84-86	82-86	81-83
Low commonality				
Average	100	87	89	81
Range	*	85-88	86-92	79-85
Medium commonality				
Average	100	86	83	77
Range	*	79-92	79-86	75-80
High commonality				
Average	100	87	93	81
Range	*	83-92	90-96	78-84

*Cost index based on Wagner and Whitin [29].

Table 5: Average 52-period results.

Cost Indices	Wagner and Whitin [29]	Rehmani and Steinberg [24]	McLaren and Whybark [17]	Total Average Modification (TAM)
Grand average of cost indices	100	90	90	82

*Cost index based on Wagner and Whitin [29].

and product structure levels. The cost savings produced by TAM for the 52-period problems were always superior to WW and were superior or equal to RS and MW in 47 out of the 48 test problems. TAM has been consistently and significantly superior to WW, MW, and RS on the general product structure, zero production/purchasing costs problem. More importantly, TAM is able to accommodate variable production/purchasing costs while other approaches in the reviewed literature cannot. TAM may allow the general product structure, variable production/purchasing costs MRP lot-sizing problems encountered in manufacturing to be solved with "good" results. [Received: March 4, 1987. Accepted: June 2, 1988.]

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